Seminar

Quantum problems of the mesoscopic physics

QUANTUM PHASE TRANSITIONS

Presentation

by

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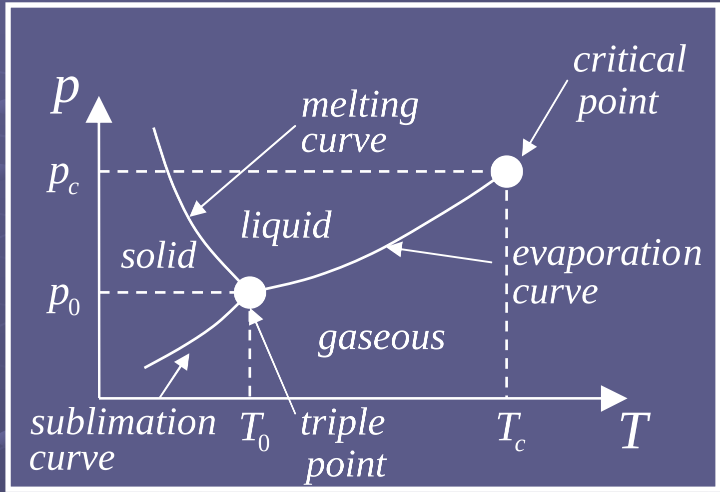
What is a phase transition?

A phase transition is a qualitative change in the properties of a system driven by the variation of an external control parameter.



[6]

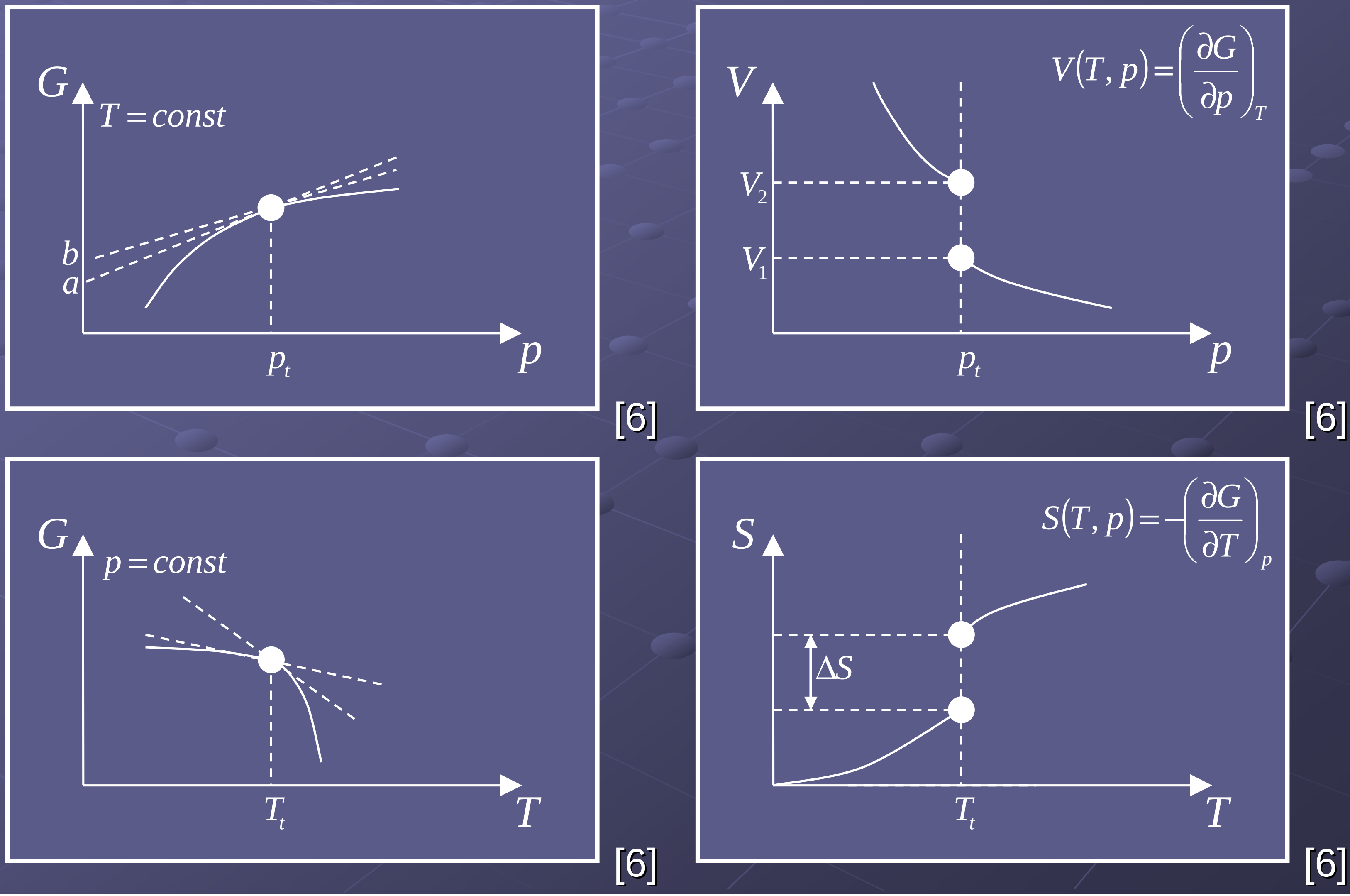
phase diagram of water

examples:

- transitions between solid, liquid and gaseous
- transition of a metal from the normal conducting to the superconducting state
- transition of a ferromagnet into the paramagnetic state at the Curie temperature

First-order phase transitions

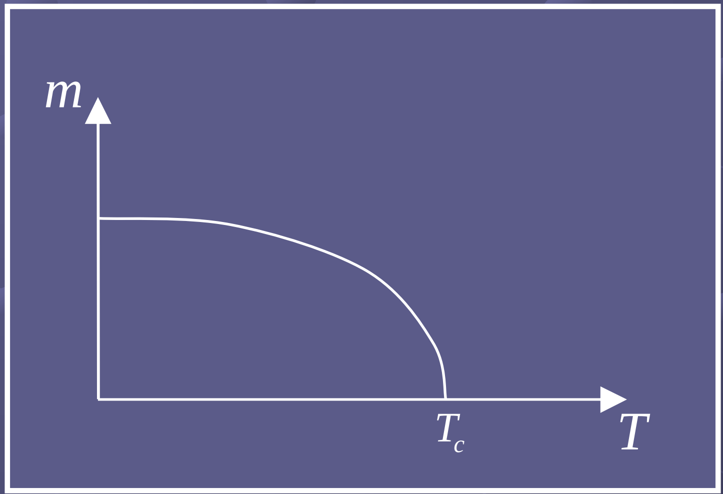
at the example of a fluid system $G(T,p) = U - TS + pV$



Continuous phase transitions

order parameter
$\neq 0$ in the ordered phase
$= 0$ in the disordered phase

example:
ferromagnetic transition of iron at 770°C
order parameter: total magnetization *m*



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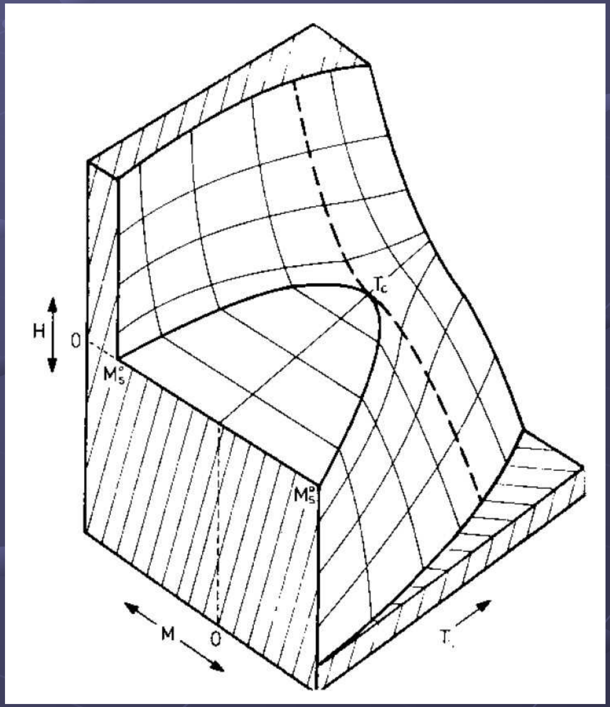

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phase diagram of a ferromagnet

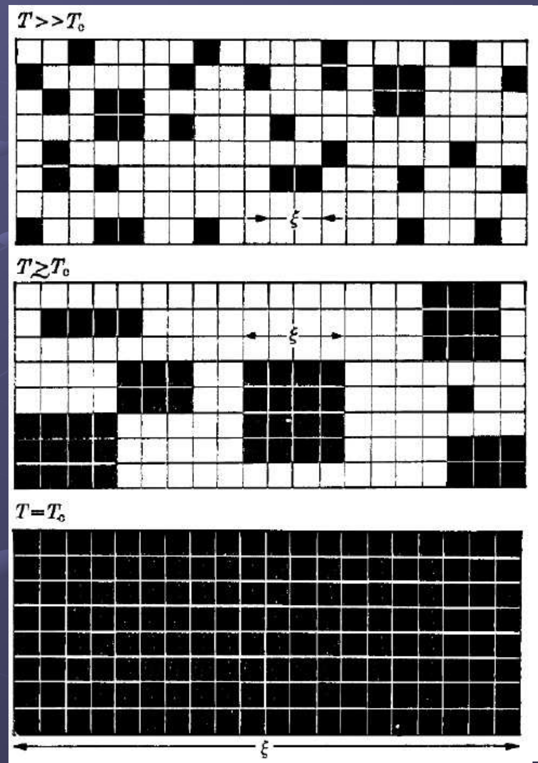
Critical phenomena

fluctuations of the order parameter $\neq 0$

typical length scale of spatial correlations
correlation length ξ

typical time scale for decay of fluctuations
correlation time τ_c

ξ and τ_c diverge in the vicinity of the critical point depending on power laws
➡ critical phenomena



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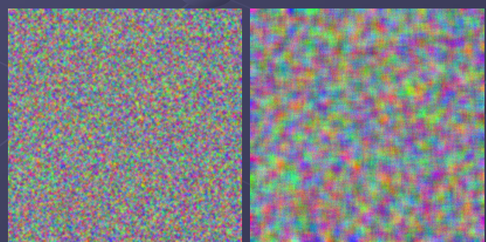
model system of localized spins

system is scale-invariant



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Critical exponents

	exponent	definition	conditions
specific heat	α	$C \propto \lvert t \rvert^{-\alpha}$	$t \to 0, B = 0$
order parameter	β	$m \propto (-t)^{\beta}$	$t \uparrow 0, B = 0$
susceptibility	γ	$\chi \propto \lvert t \rvert^{-\gamma}$	$t \to 0, B = 0$
critical isotherm	δ	$B \propto \lvert m \rvert^{\delta} sign(m)$	$B \to 0, t = 0$
correlation length	ν	$\xi \propto \lvert t \rvert^{-\nu}$	$t \to 0, B = 0$
correlation function	η	$G(r) \propto \lvert r \rvert^{-d+2-\eta}$	$t = 0, B = 0$
dynamic	z	$\tau_c \propto \xi^{z}$	$t \to 0, B = 0$

[2]

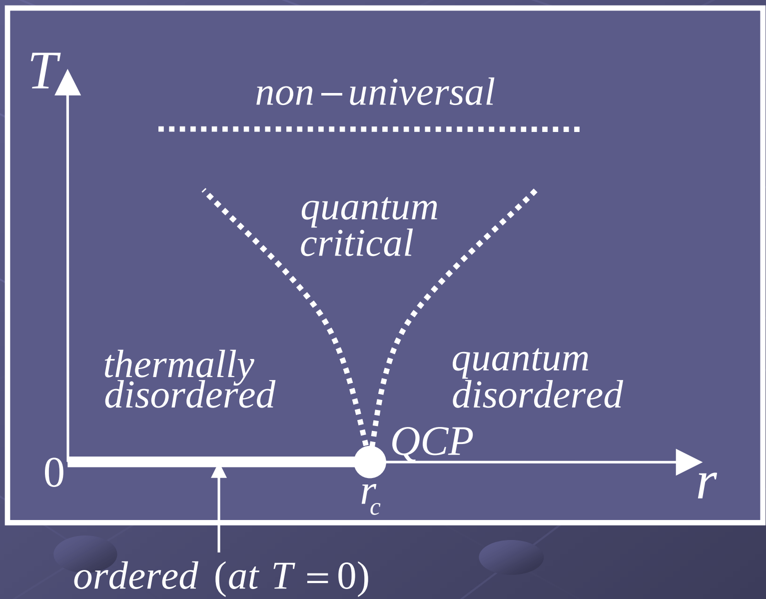
critical exponents for magnets
order parameter: magnetization *m*
conjugate field: magnetic field *B*
t distance from the critical point
d space dimensionality

universality
critical exponents are the same for entire classes of phase transitions in a wide variety of physical systems

Quantum phase transitions

occur at T=0
order destroyed by quantum fluctuations
fundamental change in the ground state of the system
caused by the variation of a non-thermal parameter *r*

Order only exists at zero temperature



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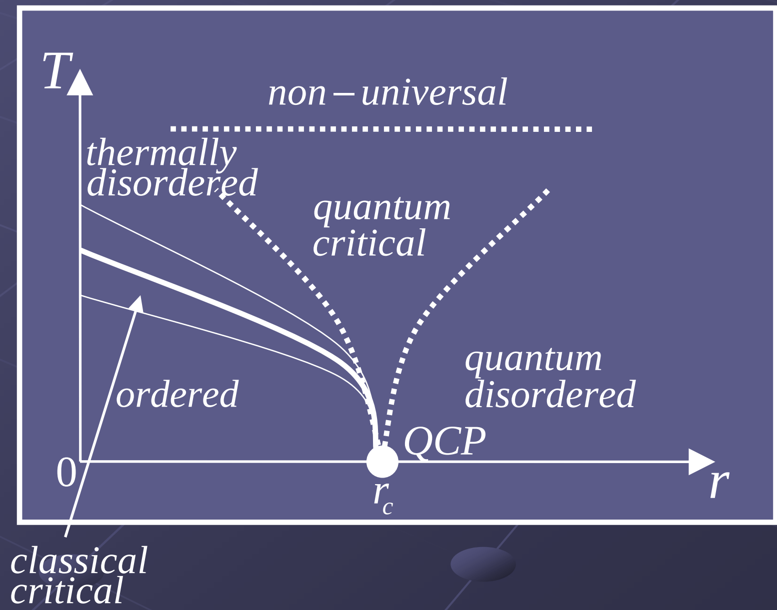
Order also exists at finite temperature



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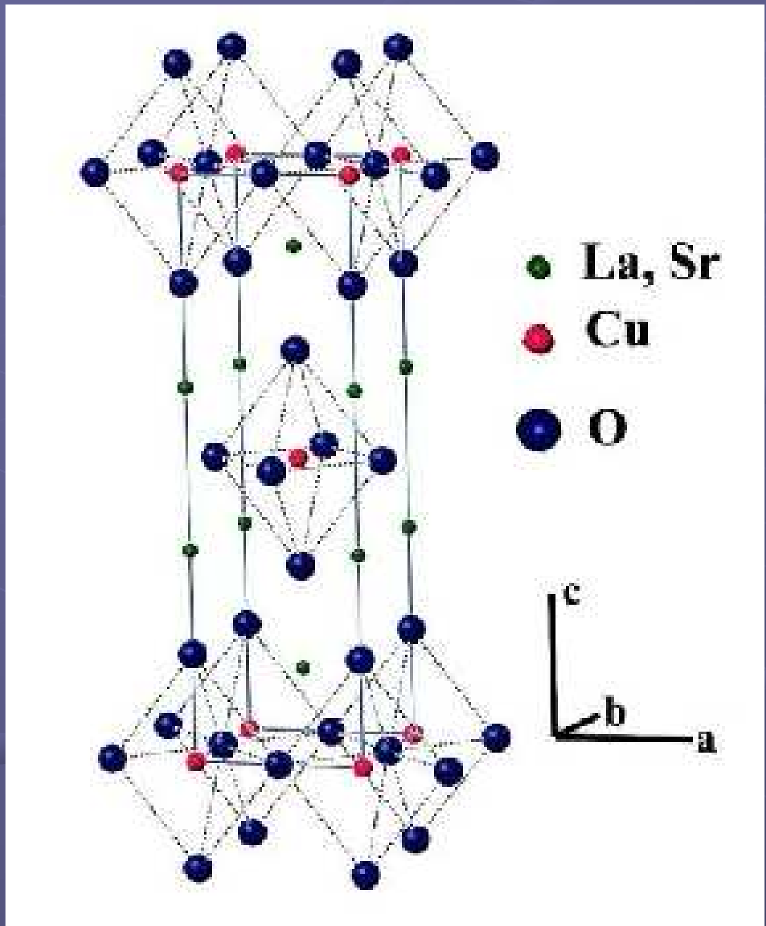
High-temperature superconductor



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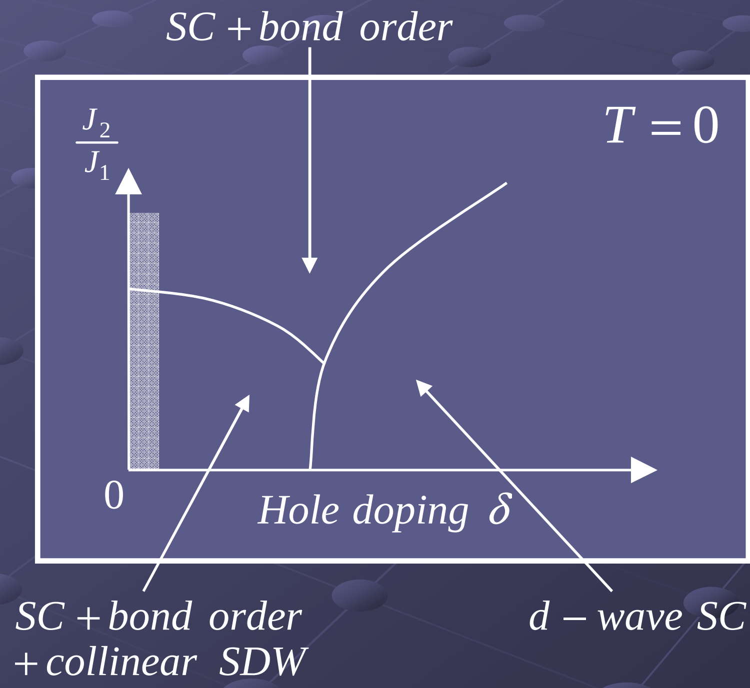
crystal lattice structure of $La_{2-x}Sr_xCuO_4$

replacement of La^{3+} by Sr^{2+}

→ creation of holes



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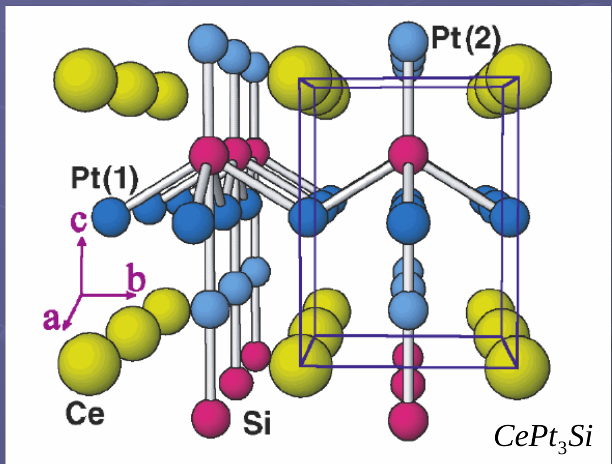
Heavy fermion compounds

= intermetallic compounds with an incomplete filled electronic f shell, like Ce, Yb or U compounds showing strongly renormalized Fermi liquid properties at low temperature



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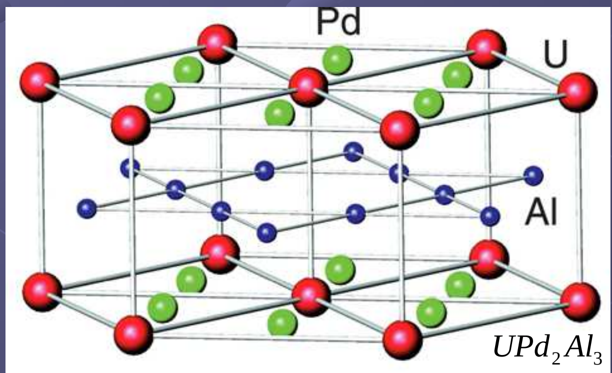

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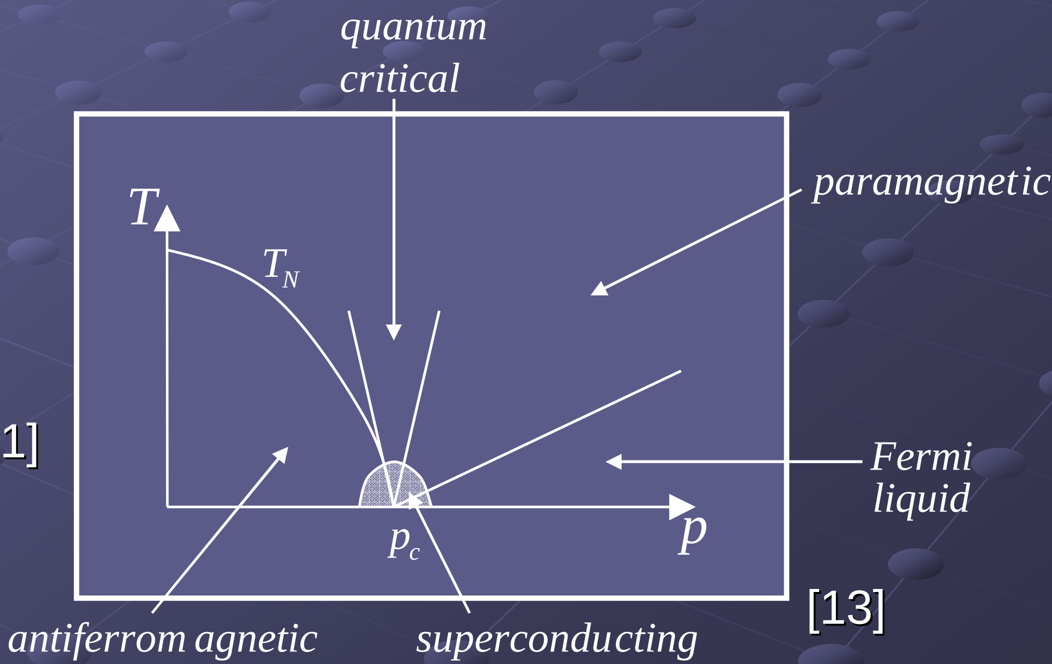

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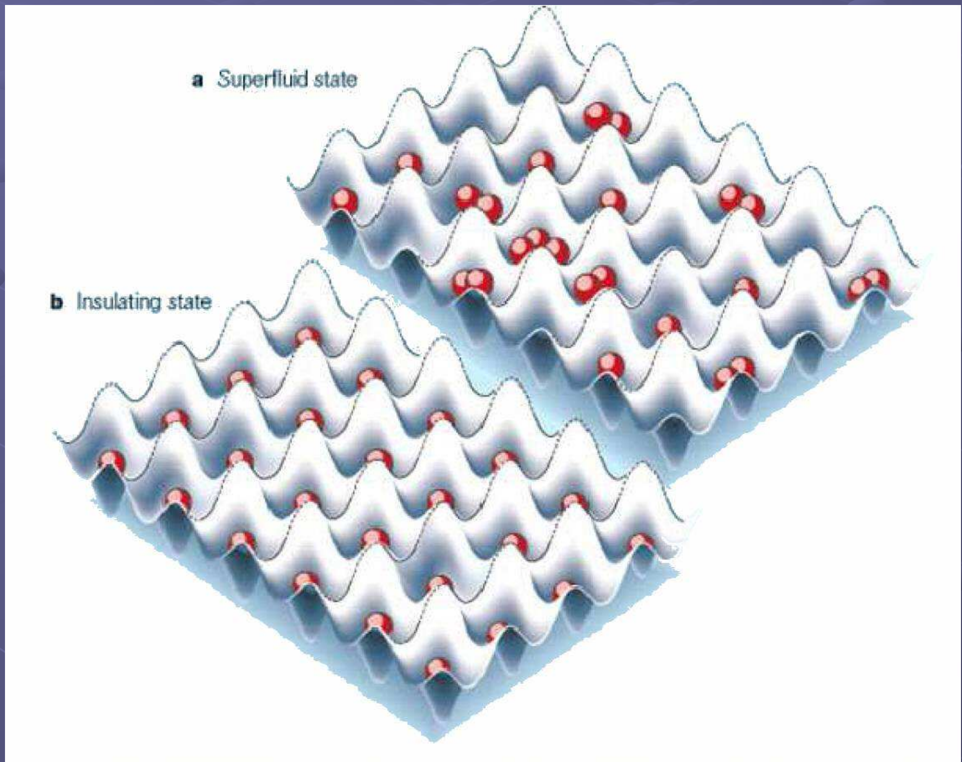
Bose-Hubbard model

describes interacting bosons on a lattice

$$\hat{H} = \frac{1}{2}\sum_{i,j}\hat{n}_i U_{ij}\hat{n}_j - \mu\sum_i \hat{n}_i - t\sum_{\langle i,j\rangle}\left(\hat{b}_i^+\hat{b}_j + \hat{b}_j^+\hat{b}_i\right)$$

examples:
- bosonic atoms on an optical lattice
- Josephson junction arrays



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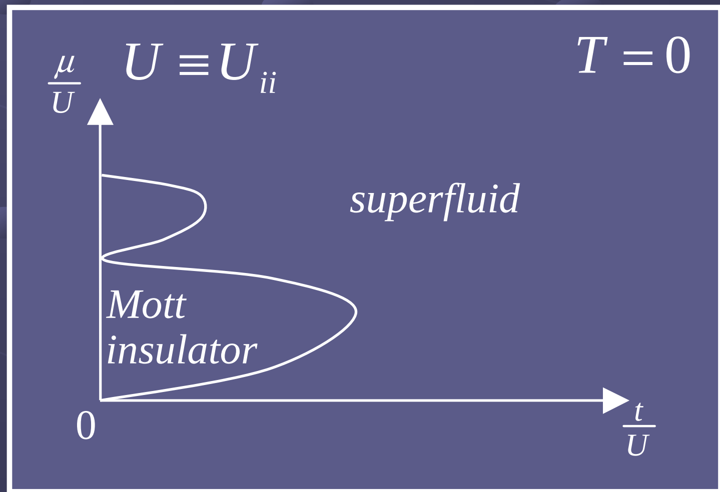

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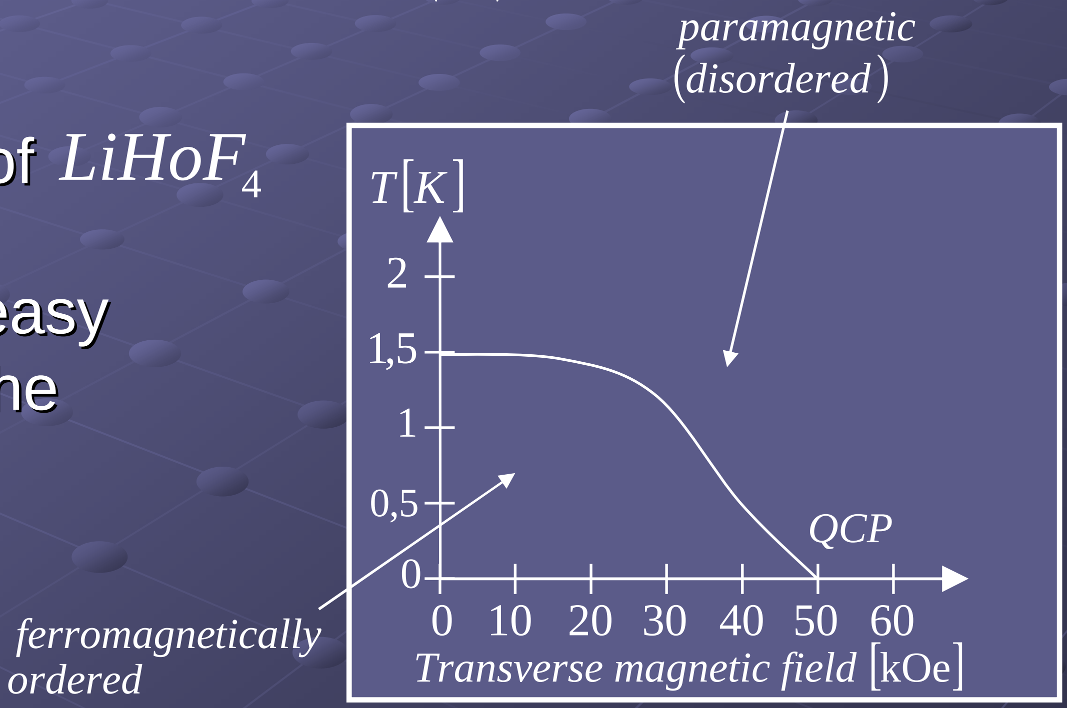
Quantum Ising model

$$\hat{H} = -g\sum_{i}\hat{\sigma}_i^x - J\sum_{\langle i,j\rangle}\hat{\sigma}_i^z\hat{\sigma}_j^z$$

example:
magnetic properties of $LiHoF_4$
at low temperatures
ionic crystal with an easy
axis for the spins of the
Holmium atoms



[2]

Quantum-classical mapping

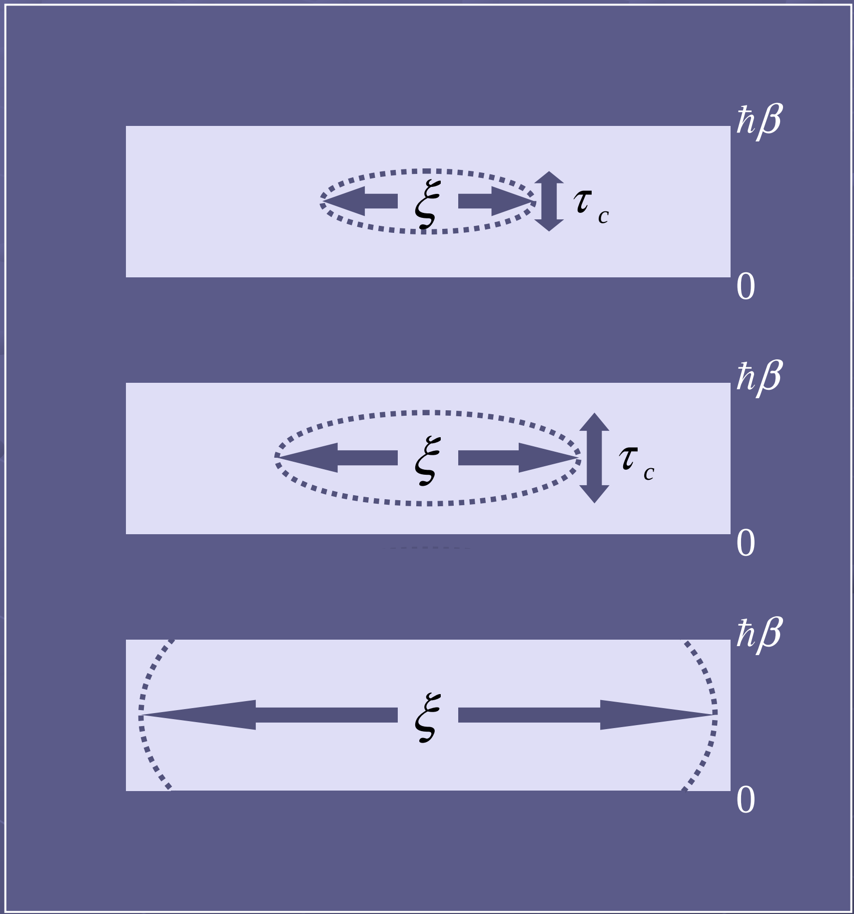
in many cases *d*-dimensional quantum systems can be mapped onto *d+1*-dimensional effective classical systems
example: quantum Ising model

size of the extra dimension determined by $\hbar\beta$

diverges for T$\rightarrow$0
$\Rightarrow$ quantum effects are always important

Quantum-classical mapping

finite for T>0



τ_c much shorter than $\hbar\beta$

τ_c comparable to $\hbar\beta$

close to the critical point $\tau_c > \hbar\beta$

➡ system realizes that it is not *d+1*-dimensional

➡ crossover from quantum to classical behaviour

[1]

Partition function

$Z(\beta) = Tr\left(e^{-\beta\hat{H}}\right)$ Hamiltonian $\hat{H} = \hat{T} + \hat{V}$

introduction of imaginary time $\tau = -i\hbar\beta$

operator density matrix $e^{-\beta\hat{H}}$ ➡ $e^{-i\hat{H}\tau/\hbar}$ time-evolution operator

➡ introduction of an imaginary time direction using path integral formalism

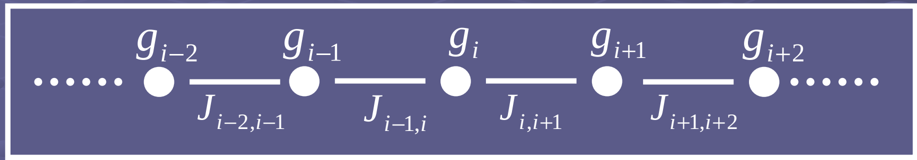
$\hat{T}$ and $\hat{V}$ don't commute in general

➡ $Z(\beta)$ doesn't factorize

➡ statics and dynamics of a quantum system are always coupled

Disordered systems

disordered quantum Ising chain



Hamiltonian for 1-dimensional system

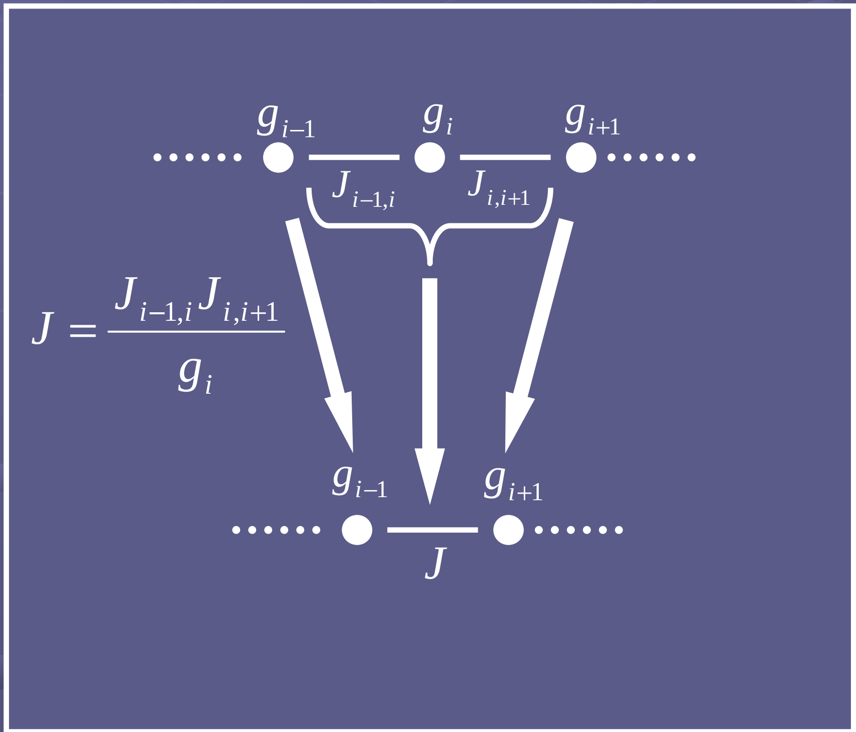
$$\hat{H} = -\sum_i g_i \hat{\sigma}_i^x - \sum_i J_{i,i+1} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

couplings vary from point to point

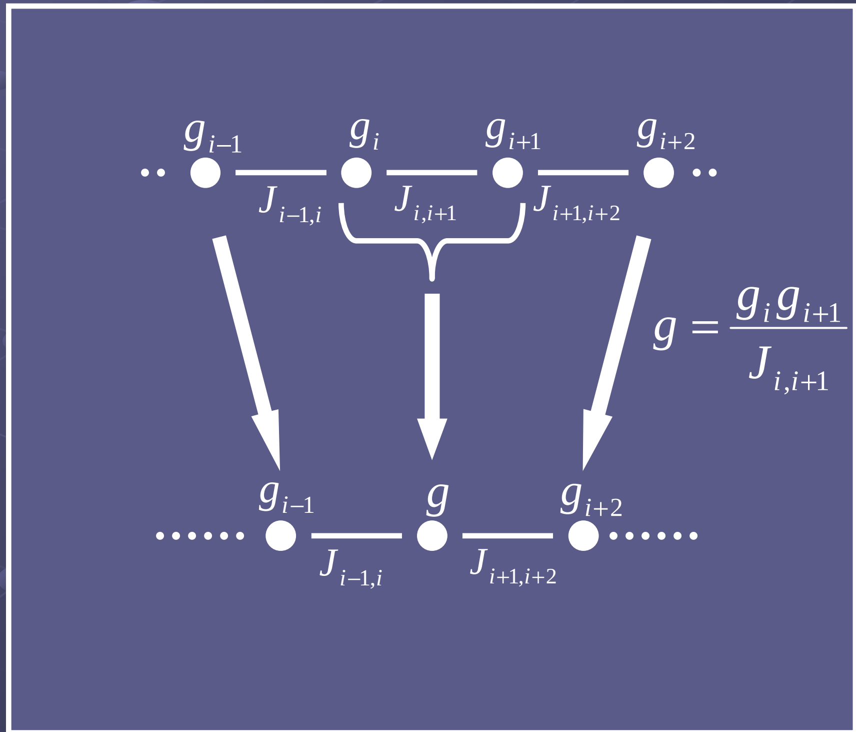
general distribution of fields g_i and bonds $J_{i,i+1}$

Renormalization group analysis

Maximum coupling is a field



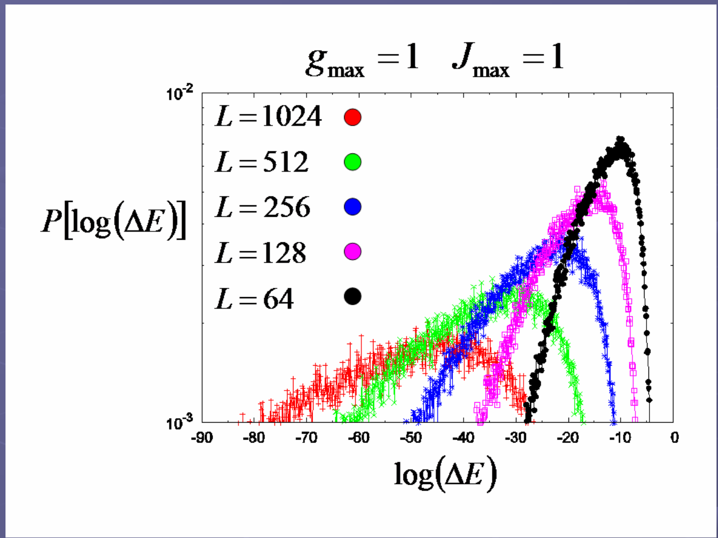
Maximum coupling is a bond



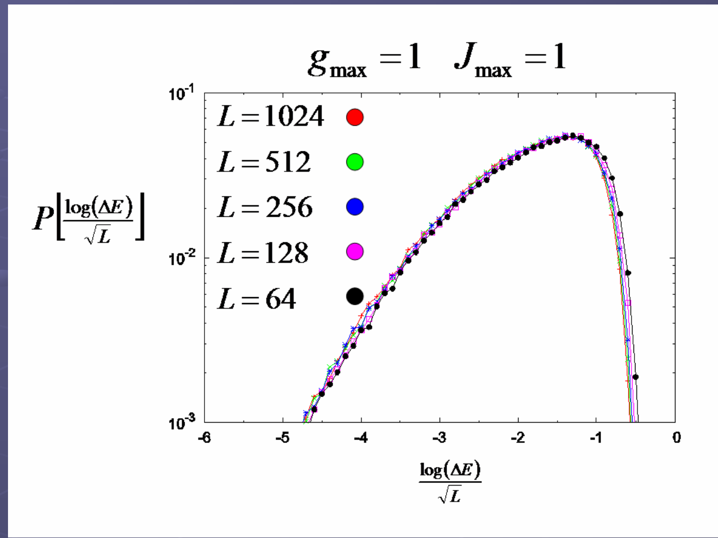
Renormalization group analysis

- decimation procedure is the basic renormalization group transformation
- each step reduces the number of the sites and the number of the bonds by one
- iteration of the transformation till maximum remaining coupling is of the order of the energy at which the system is probed
- different bonds and fields remain independent random variables
- probability distributions are renormalized

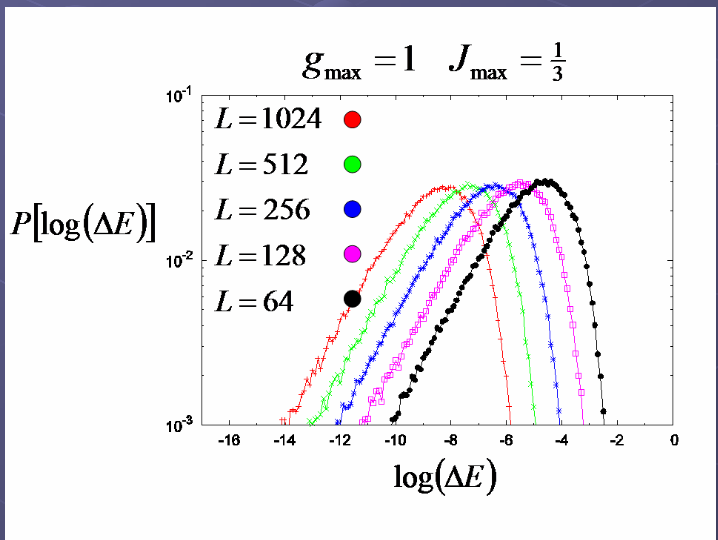
Results



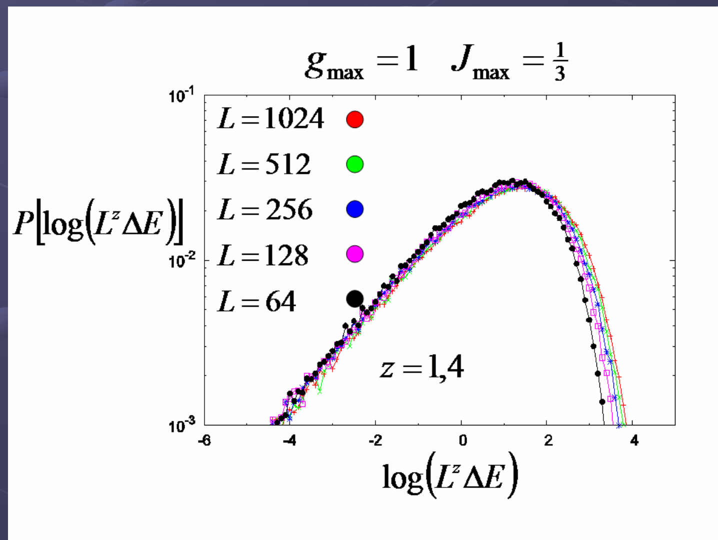
$g_{max} = 1 \quad J_{max} = 1$
$P[\log(\Delta E)]$
$L = 1024$
$L = 512$
$L = 256$
$L = 128$
$L = 64$
10^{-2}
10^{-3}
-90 -80 -70 -60 -50 -40 -30 -20 -10 0
$\log(\Delta E)$



$g_{max} = 1 \quad J_{max} = 1$
$P\left[\frac{\log(\Delta E)}{\sqrt{L}}\right]$
$L = 1024$
$L = 512$
$L = 256$
$L = 128$
$L = 64$
10^{-1}
10^{-2}
10^{-3}
-6 -5 -4 -3 -2 -1 0
$\frac{\log(\Delta E)}{\sqrt{L}}$

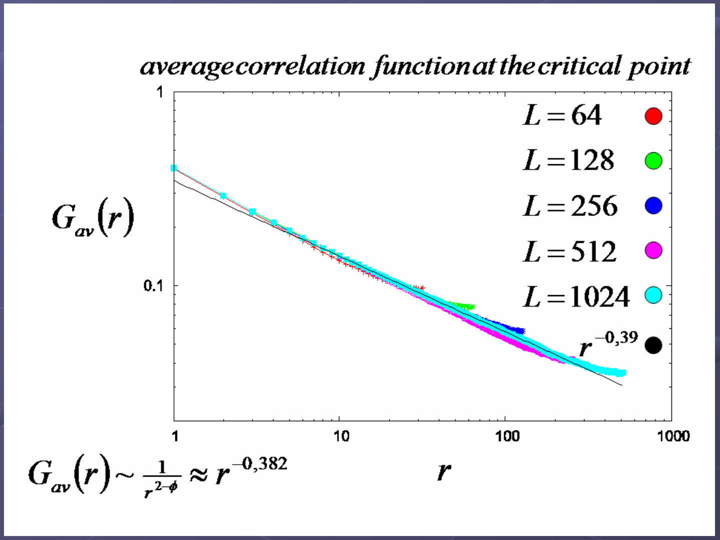


$g_{max} = 1 \quad J_{max} = \frac{1}{3}$
$P[\log(\Delta E)]$
$L = 1024$
$L = 512$
$L = 256$
$L = 128$
$L = 64$
10^{-1}
10^{-2}
10^{-3}
-16 -14 -12 -10 -8 -6 -4 -2 0
$\log(\Delta E)$



$g_{max} = 1 \quad J_{max} = \frac{1}{3}$
$P[\log(L^z \Delta E)]$
$L = 1024$
$L = 512$
$L = 256$
$L = 128$
$L = 64$
$z = 1{,}4$
10^{-1}
10^{-2}
10^{-3}
-6 -4 -2 0 2 4
$\log(L^z \Delta E)$

Results



average correlation function at the critical point
1
0.1
$G_{av}(r)$
$L=64$
$L=128$
$L=256$
$L=512$
$L=1024$
$r^{-0,39}$
1
10
100
1000
$G_{av}(r) \sim \frac{1}{r^{2-\phi}} \approx r^{-0,382}$
r

Summary

quantum phase transitions occur at T=0

critical exponents control the behaviour of the system in the vicinity of the critical point

systems may be assigned to different universality classes with common critical exponents

in many cases *d*-dimensional quantum systems can be mapped onto *d+1*-dimensional effective classical systems

for a quantum system statics and dynamics are always coupled

References

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